Character tables for some chemically important symmetry groups

C_s	E	σ_h		
A′	1	1	T_x, T_y, R_z	x^2, y^2, z^2, xy
A″	1	−1	T_z, R_x, R_y	yz, zx

C_i	E	i		
A_g	1	1	R_x, R_y, R_z	x^2, y^2, z^2, xy, zx, yz
A_u	1	−1	T_x, T_y, T_z	

The C_n groups

C_2	E	C_2		
A	1	1	$T_z, R_z,$	x^2, y^2, z^2, xy
B	1	−1	T_x, T_y, R_x, R_y	yz, zx

C_3	E	C_3	C_3^2		$\epsilon = \exp(2\pi i/3)$
A	1	1	1	T_z, R_z	$x^2 + y^2, z^2$
E	1	ϵ	ϵ^*	$(T_x, T_y), (R_x, R_y)$	$(x^2 - y^2, xy), (yz, zx)$
	1	ϵ^*	ϵ		

C_4	E	C_4	C_2	C_4^3		
A	1	1	1	1	T_z, R_z	$x^2 + y^2, z^2$
B	1	−1	1	−1		$x^2 - y^2, xy$
E	1	i	−1	$-i$	$(T_x, T_y), (R_x, R_y)$	(yz, zx)
	1	$-i$	−1	i		

C_5	E	C_5	C_5^2	C_5^3	C_5^4		$\epsilon = \exp(2\pi i/5)$
A	1	1	1	1	1	T_z, R_z	$x^2 + y^2, z^2$
E_1	1	ϵ	ϵ^2	ϵ^{2*}	ϵ^*	$(T_x, T_y), (R_x, R_y)$	(yz, zx)
	1	ϵ^*	ϵ^{2*}	ϵ^2	ϵ		
E_2	1	ϵ^2	ϵ^*	ϵ	ϵ^{2*}		$(x^2 - y^2, xy)$
	1	ϵ^{2*}	ϵ	ϵ^*	ϵ^2		

C_6	E	C_6	C_3	C_2	C_3^2	C_6^5		$\epsilon = \exp(2\pi i/6)$
A	1	1	1	1	1	1	T_z, R_z	$x^2 + y^2, z^2$
B	1	−1	1	−1	1	−1		
E_1	1	ϵ	$-\epsilon^*$	−1	$-\epsilon$	ϵ^*	(T_x, T_y), (R_x, R_y)	(yz, zx)
	1	ϵ^*	$-\epsilon$	−1	$-\epsilon^*$	ϵ		
E_2	1	$-\epsilon^*$	$-\epsilon$	1	$-\epsilon^*$	$-\epsilon$		$(x^2 - y^2, xy)$
	1	$-\epsilon$	$-\epsilon^*$	1	$-\epsilon$	$-\epsilon^*$		

C_7	E	C_7	C_7^2	C_7^3	C_7^4	C_7^5	C_7^6		$\epsilon = \exp(2\pi i/7)$
A	1	1	1	1	1	1	1	T_z, R_z	$x^2 + y^2, z^2$
E_1	1	ϵ	ϵ^2	ϵ^3	ϵ^{3*}	ϵ^{2*}	ϵ^*	(T_x, T_y)	(yz, zx)
	1	ϵ^*	ϵ^{2*}	ϵ^{3*}	ϵ^3	ϵ^2	ϵ	(R_x, R_y)	
E_2	1	ϵ^2	ϵ^{3*}	ϵ^*	ϵ	ϵ^3	ϵ^{2*}		$(x^2 - y^2, xy)$
	1	ϵ^{2*}	ϵ^3	ϵ	ϵ^*	ϵ^{3*}	ϵ^2		
E_3	1	ϵ^3	ϵ^*	ϵ^2	ϵ^{2*}	ϵ	ϵ^{3*}		
	1	ϵ^{3*}	ϵ	ϵ^{2*}	ϵ^2	ϵ^*	ϵ^3		

C_8	E	C_8	C_4	C_2	C_4^3	C_8^3	C_8^5	C_8^7		$\epsilon = \exp(2\pi i/8)$
A	1	1	1	1	1	1	1	1	T_z, R_z	$x^2 + y^2, z^2$
B	1	−1	1	1	1	−1	−1	−1		
E_1	1	ϵ	i	−1	$-i$	$-\epsilon^*$	$-\epsilon$	ϵ^*	(T_x, T_y), (R_x, R_y)	(yz, zx)
	1	ϵ^*	$-i$	−1	i	$-\epsilon$	$-\epsilon^*$	ϵ		
E_2	1	i	−1	1	−1	$-i$	i	$-i$		$(x^2 - y^2, xy)$
	1	$-i$	−1	1	−1	i	$-i$	i		
E_3	1	$-\epsilon$	i	−1	$-i$	ϵ^*	ϵ	$-\epsilon^*$		
	1	$-\epsilon^*$	$-i$	−1	i	ϵ	ϵ^*	$-\epsilon$		

The D_n groups

D_2	E	$C_2(z)$	$C_2(y)$	$C_2(x)$		
A	1	1	1	1		x^2, y^2, z^2
B_1	1	1	−1	−1	T_z, R_z	xy
B_2	1	−1	1	−1	T_y, R_y	zx
B_3	1	−1	−1	1	T_x, R_x	yz

D_3	E	$2C_3$	$3C_2$		
A_1	1	1	1		$x^2 + y^2, z^2$
A_2	1	1	−1	T_z, R_z	
E	2	−1	0	$(T_x, T_y), (R_x, R_y)$	$(x^2 - y^2, xy), (yz, zx)$

D_4	E	$2C_4$	$C_2(=C_4^2)$	$2C_2'$	$2C_2''$		
A_1	1	1	1	1	1		$x^2 + y^2, z^2$
A_2	1	1	1	−1	−1	T_z, R_z	
B_1	1	−1	1	1	−1		$x^2 - y^2$
B_2	1	−1	1	−1	1		xy
E	2	0	−2	0	0	$(T_x, T_y), (R_x, R_y)$	(yz, zx)

$\mathbf{D}_5$	E	$2C_5$	$2C_5^2$	$5C_2$		
A_1	1	1	1	1		$x^2 + y^2, z^2$
A_2	1	1	1	−1	$\mathbf{T}_z, \mathbf{R}_z$	
E_1	2	2 cos 72°	2 cos 144°	0	$(\mathbf{T}_x, \mathbf{T}_y), (\mathbf{R}_x, \mathbf{R}_y)$	(yz, zx)
E_2	2	2 cos 144°	2 cos 72°	0		$(x^2 - y^2, xy)$

$\mathbf{D}_6$	E	$2C_6$	$2C_3$	C_2	$3C_2'$	$3C_2''$		
A_1	1	1	1	1	1	1		$x^2 + y^2, z^2$
A_2	1	1	1	1	−1	−1	$\mathbf{T}_z, \mathbf{R}_z$	
B_1	1	−1	1	−1	1	−1		
B_2	1	−1	1	−1	−1	1		
E_1	2	1	−1	−2	0	0	$(\mathbf{T}_x, \mathbf{T}_y), (\mathbf{R}_x, \mathbf{R}_y)$	(yz, zx)
E_2	2	−1	−1	2	0	0		$(x^2 - y^2, xy)$

The $\mathbf{C}_{nv}$ groups

$\mathbf{C}_{2v}$	E	C_2	$\sigma_v(xz)$	$\sigma_v'(yz)$		
A_1	1	1	1	1	$\mathbf{T}_z$	x^2, y^2, z^2
A_2	1	1	−1	−1	$\mathbf{R}_z$	xy
B_1	1	−1	1	−1	$\mathbf{T}_x, \mathbf{R}_y$	zx
B_2	1	−1	−1	1	$\mathbf{T}_y, \mathbf{R}_x$	yz

$\mathbf{C}_{3v}$	E	$2C_3$	$3\sigma_v$		
A_1	1	1	1	$\mathbf{T}_z$	$x^2 + y^2, z^2$
A_2	1	1	−1	$\mathbf{R}_z$	
E	2	−1	0	$(\mathbf{T}_x, \mathbf{T}_y); (\mathbf{R}_x, \mathbf{R}_y)$	$(x^2 - y^2, xy), (yz, zx)$

$\mathbf{C}_{4v}$	E	$2C_4$	C_2	$2\sigma_v$	$2\sigma_d$		
A_1	1	1	1	1	1	$\mathbf{T}_z$	$x^2 + y^2, z^2$
A_2	1	1	1	−1	−1	$\mathbf{R}_z$	
B_1	1	−1	1	1	−1		$x^2 - y^2$
B_2	1	−1	1	−1	1		xy
E	2	0	−2	0	0	$(\mathbf{T}_x, \mathbf{T}_y), (\mathbf{R}_x, \mathbf{R}_y)$	(yz, zx)

$\mathbf{C}_{5v}$	E	$2C_5$	$2C_5^2$	$5\sigma_v$		
A_1	1	1	1	1	$\mathbf{T}_z$	$x^2 + y^2, z^2$
A_2	1	1	1	−1	$\mathbf{R}_z$	
E_1	2	2 cos 72°	2 cos 144°	0	$(\mathbf{T}_x, \mathbf{T}_y), (\mathbf{R}_x, \mathbf{R}_y)$	(yz, zx)
E_2	2	2 cos 144°	2 cos 72°	0		$(x^2 - y^2, xy)$

$\mathbf{C}_{6v}$	E	$2C_6$	$2C_3$	C_2	$3\sigma_v$	$3\sigma_d$		
A_1	1	1	1	1	1	1	$\mathbf{T}_z$	$x^2 + y^2, z^2$
A_2	1	1	1	1	−1	−1	$\mathbf{R}_z$	
B_1	1	−1	1	−1	1	−1		
B_2	1	−1	1	−1	−1	1		
E_1	2	1	−1	−2	0	0	$(\mathbf{T}_x, \mathbf{T}_y), (\mathbf{R}_x, \mathbf{R}_y)$	(yz, zx)
E_2	2	−1	−1	2	0	0		$(x^2 - y^2, xy)$

The $\mathbf{C}_{nh}$ groups

$\mathbf{C}_{2h}$	E	C_2	i	σ_h		
A_g	1	1	1	1	$\mathbf{R}_z$	x^2, y^2, z^2, xy
B_g	1	−1	1	−1	$\mathbf{R}_x, \mathbf{R}_y$	yz, zx
A_u	1	1	−1	−1	$\mathbf{T}_z$	
B_u	1	−1	−1	1	$\mathbf{T}_x, \mathbf{T}_y$	

$\mathbf{C}_{3h}$	E	C_3	C_3^2	σ_h	S_3	S_3^2		$\epsilon = \exp(2\pi i/3)$
A'	1	1	1	1	1	1	$\mathbf{R}_z$	$x^2 + y^2, z^2$
E'	1	ϵ	ϵ^*	1	ϵ	ϵ^*	$(\mathbf{T}_x, \mathbf{T}_y)$	$(x^2 - y^2, xy)$
	1	ϵ^*	ϵ	1	ϵ^*	ϵ		
A''	1	1	1	−1	−1	−1	$\mathbf{T}_z$	
E''	1	ϵ	ϵ^*	−1	$-\epsilon$	$-\epsilon^*$	$(\mathbf{R}_x, \mathbf{R}_y)$	(yz, zx)
	1	ϵ^*	ϵ	−1	$-\epsilon^*$	$-\epsilon$		

$\mathbf{C}_{4h}$	E	C_4	C_2	C_4^3	i	S_4^3	σ_h	S_4		
A_g	1	1	1	1	1	1	1	1	$\mathbf{R}_z$	$x^2 + y^2, z^2$
B_g	1	−1	1	−1	1	−1	1	−1		$x^2 - y^2, xy$
E_g	1	i	−1	$-i$	1	i	−1	$-i$	$(\mathbf{R}_x, \mathbf{R}_y)$	(yz, zx)
	1	$-i$	−1	i	1	$-i$	−1	i		
A_u	1	1	1	1	−1	−1	−1	−1	$\mathbf{T}_z$	
B_u	1	−1	1	−1	−1	1	−1	1		
E_u	1	i	−1	$-i$	−1	$-i$	1	i	$(\mathbf{T}_x, \mathbf{T}_y)$	
	1	$-i$	−1	i	−1	i	1	$-i$		

$\mathbf{C}_{5h}$	E	C_5	C_5^2	C_5^3	C_5^4	σ_h	S_5	S_5^7	S_5^3	S_5^9		$\epsilon = \exp(2\pi i/5)$
A'	1	1	1	1	1	1	1	1	1	1	$\mathbf{R}_z$	$x^2 + y^2, z^2$
E_1'	1	ϵ	ϵ^2	ϵ^{2*}	ϵ^*	1	ϵ	ϵ^2	ϵ^{2*}	ϵ^*	$(\mathbf{T}_x, \mathbf{T}_y)$	
	1	ϵ^*	ϵ^{2*}	ϵ^2	ϵ	1	ϵ^*	ϵ^{2*}	ϵ^2	ϵ		
E_2'	1	ϵ^2	ϵ^*	ϵ	ϵ^{2*}	1	ϵ^2	ϵ^*	ϵ	ϵ^{2*}		$(x^2 - y^2, xy)$
	1	ϵ^{2*}	ϵ	ϵ^*	ϵ^2	1	ϵ^{2*}	ϵ	ϵ^*	ϵ^2		
A''	1	1	1	1	1	−1	−1	−1	−1	−1	$\mathbf{T}_z$	
E_1''	1	ϵ	ϵ^2	ϵ^{2*}	ϵ^*	−1	$-\epsilon$	$-\epsilon^2$	$-\epsilon^{2*}$	$-\epsilon^*$	$(\mathbf{R}_x, \mathbf{R}_y)$	(yz, zx)
	1	ϵ^*	ϵ^{2*}	ϵ^2	ϵ	−1	$-\epsilon^*$	$-\epsilon^{2*}$	$-\epsilon^2$	$-\epsilon$		
E_2''	1	ϵ^2	ϵ^*	ϵ	ϵ^{2*}	−1	$-\epsilon^2$	$-\epsilon^*$	$-\epsilon$	$-\epsilon^{2*}$		
	1	ϵ^{2*}	ϵ	ϵ^*	ϵ^2	−1	$-\epsilon^{2*}$	$-\epsilon$	$-\epsilon^*$	$-\epsilon^2$		

$\mathbf{C}_{6h}$	E	C_6	C_3	C_2	C_3^2	C_6^5	i	S_3^5	S_6^5	σ_h	S_6	S_3		$\epsilon = \exp(2\pi i/6)$
A_g	1	1	1	1	1	1	1	1	1	1	1	1	$\mathbf{R}_z$	$x^2 + y^2, z^2$
B_g	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1		
E_{1g}	1	ϵ	$-\epsilon^*$	−1	$-\epsilon$	ϵ^*	1	ϵ	$-\epsilon^*$	−1	$-\epsilon$	ϵ^*	$(\mathbf{R}_x, \mathbf{R}_y)$	(yz, zx)
	1	ϵ^*	$-\epsilon$	−1	$-\epsilon^*$	ϵ	1	ϵ^*	$-\epsilon$	−1	$-\epsilon^*$	ϵ		
E_{2g}	1	$-\epsilon^*$	$-\epsilon$	1	$-\epsilon^*$	$-\epsilon$	1	$-\epsilon^*$	$-\epsilon$	1	$-\epsilon^*$	$-\epsilon$		$(x^2 - y^2, xy)$
	1	$-\epsilon$	$-\epsilon^*$	1	$-\epsilon$	$-\epsilon^*$	1	$-\epsilon$	$-\epsilon^*$	1	$-\epsilon$	$-\epsilon^*$		
A_u	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	$\mathbf{T}_z$	
B_u	1	−1	1	−1	1	−1	−1	1	−1	1	−1	1		
E_{1u}	1	ϵ	$-\epsilon^*$	−1	$-\epsilon$	ϵ^*	−1	$-\epsilon$	ϵ^*	1	ϵ	$-\epsilon^*$	$(\mathbf{T}_x, \mathbf{T}_y)$	
	1	ϵ^*	$-\epsilon$	−1	$-\epsilon^*$	ϵ	−1	$-\epsilon^*$	ϵ	1	ϵ^*	$-\epsilon$		
E_{2u}	1	$-\epsilon^*$	$-\epsilon$	1	$-\epsilon^*$	$-\epsilon$	−1	ϵ^*	ϵ	−1	ϵ^*	ϵ		
	1	$-\epsilon$	$-\epsilon^*$	1	$-\epsilon$	$-\epsilon^*$	−1	ϵ	ϵ^*	−1	ϵ	ϵ^*		

The D_{nh} groups

D_{2h}	E	$C_2(z)$	$C_2(y)$	$C_2(x)$	i	$\sigma(xy)$	$\sigma(xz)$	$\sigma(yz)$		
A_g	1	1	1	1	1	1	1	1		x^2, y^2, z^2
B_{1g}	1	1	−1	−1	1	1	−1	−1	$\mathbf{R_z}$	xy
B_{2g}	1	−1	1	−1	1	−1	1	−1	$\mathbf{R_y}$	zx
B_{3g}	1	−1	−1	1	1	−1	−1	1	$\mathbf{R_x}$	yz
A_u	1	1	1	1	−1	−1	−1	−1		
B_{1u}	1	1	−1	−1	−1	−1	1	1	$\mathbf{T_z}$	
B_{2u}	1	−1	1	−1	−1	1	−1	1	$\mathbf{T_y}$	
B_{3u}	1	−1	−1	1	−1	1	1	−1	$\mathbf{T_x}$	

D_{3h}	E	$2C_3$	$3C_2$	σ_h	$2S_3$	$3\sigma_v$		
A_1'	1	1	1	1	1	1		$x^2 + y^2, z^2$
A_2'	1	1	−1	1	1	−1	$\mathbf{R_z}$	
E'	2	−1	0	2	−1	0	$(\mathbf{T_x}, \mathbf{T_y})$	$(x^2 - y^2, xy)$
A_1''	1	1	1	−1	−1	−1		
A_2''	1	1	−1	−1	−1	1	$\mathbf{T_z}$	
E''	2	−1	0	−2	1	0	$(\mathbf{R_x}, \mathbf{R_y})$	(yz, zx)

D_{4h}	E	$2C_4$	C_2	$2C_2'$	$2C_2''$	i	$2S_4$	σ_h	$2\sigma_v$	$2\sigma_d$		
A_{1g}	1	1	1	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
A_{2g}	1	1	1	−1	−1	1	1	1	−1	−1	$\mathbf{R_z}$	
B_{1g}	1	−1	1	1	−1	1	−1	1	1	−1		$x^2 - y^2$
B_{2g}	1	−1	1	−1	1	1	−1	1	−1	1		xy
E_g	2	0	−2	0	0	2	0	−2	0	0	$(\mathbf{R_x}, \mathbf{R_y})$	(yz, zx)
A_{1u}	1	1	1	1	1	−1	−1	−1	−1	−1		
A_{2u}	1	1	1	−1	−1	−1	−1	−1	1	1	$\mathbf{T_z}$	
B_{1u}	1	−1	1	1	−1	−1	1	−1	−1	1		
B_{2u}	1	−1	1	−1	1	−1	1	−1	1	−1		
E_u	2	0	−2	0	0	−2	0	2	0	0	$(\mathbf{T_x}, \mathbf{T_y})$	

D_{5h}	E	$2C_5$	$2C_5^2$	$5C_2$	σ_h	$2S_5$	$2S_5^3$	$5\sigma_v$		
A_1'	1	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
A_2'	1	1	1	−1	1	1	1	−1	$\mathbf{R_z}$	
E_1'	2	2 cos 72°	2 cos 144°	0	2	2 cos 72°	2 cos 144°	0	$(\mathbf{T_x}, \mathbf{T_y})$	
E_2'	2	2 cos 144°	2 cos 72°	0	2	2 cos 144°	2 cos 72°	0		$(x^2 - y^2, xy)$
A_1''	1	1	1	1	−1	−1	−1	−1		
A_2''	1	1	1	−1	−1	−1	−1	1	$\mathbf{T_z}$	
E_1''	2	2 cos 72°	2 cos 144°	0	−2	−2 cos 72°	−2 cos 144°	0	$(\mathbf{R_x}, \mathbf{R_y})$	(yz, zx)
E_2''	2	2 cos 144°	2 cos 72°	0	−2	−2 cos 144°	−2 cos 72°	0		

D_{6h}	E	$2C_6$	$2C_3$	C_2	$3C_2'$	$3C_2''$	i	$2S_3$	$2S_6$	σ_h	$3\sigma_d$	$3\sigma_v$		
A_{1g}	1	1	1	1	1	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
A_{2g}	1	1	1	1	−1	−1	1	1	1	1	−1	−1	$\mathbf{R_z}$	
B_{1g}	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1		
B_{2g}	1	−1	1	−1	−1	1	1	−1	1	−1	−1	1		
E_{1g}	2	1	−1	−2	0	0	2	1	−1	−2	0	0	$(\mathbf{R_x}, \mathbf{R_y})$	(yz, zx)
E_{2g}	2	−1	−1	2	0	0	2	−1	−1	2	0	0		$(x^2 - y^2, xy)$
A_{1u}	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1		
A_{2u}	1	1	1	1	−1	−1	−1	−1	−1	−1	1	1	$\mathbf{T_z}$	
B_{1u}	1	−1	1	−1	1	−1	−1	1	−1	1	−1	1		
B_{2u}	1	−1	1	−1	−1	1	−1	1	−1	1	1	−1		
E_{1u}	2	1	−1	−2	0	0	−2	−1	1	2	0	0	$(\mathbf{T_x}, \mathbf{T_y})$	
E_{2u}	2	−1	−1	2	0	0	−2	1	1	−2	0	0		

The D_{nd} groups

D_{2d}	E	$2S_4$	C_2	$2C_2'$	$2\sigma_d$		
A_1	1	1	1	1	1		$x^2 + y^2, z^2$
A_2	1	1	1	−1	−1	$\mathbf{R_z}$	
B_1	1	−1	1	1	−1		$x^2 - y^2$
B_2	1	−1	1	−1	1	$\mathbf{T_z}$	xy
E	2	0	−2	0	0	$(\mathbf{T_x}, \mathbf{T_y}), (\mathbf{R_x}, \mathbf{R_y})$	(yz, zx)

D_{3d}	E	$2C_3$	$3C_2$	i	$2S_6$	$3\sigma_d$		
A_{1g}	1	1	1	1	1	1		$x^2 + y^2, z^2$
A_{2g}	1	1	−1	1	1	−1	$\mathbf{R_z}$	
E_g	2	−1	0	2	−1	0	$(\mathbf{R_x}, \mathbf{R_y})$	$(x^2 - y^2, xy), (yz, zx)$
A_{1u}	1	1	1	−1	−1	−1		
A_{2u}	1	1	−1	−1	−1	1	$\mathbf{T_z}$	
E_u	2	−1	0	−2	1	0	$(\mathbf{T_x}, \mathbf{T_y})$	

D_{4d}	E	$2S_8$	$2C_4$	$2S_8^3$	C_2	$4C_2'$	$4\sigma_d$		
A_1	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
A_2	1	1	1	1	1	−1	−1	$\mathbf{R_z}$	
B_1	1	−1	1	−1	1	1	−1		
B_2	1	−1	1	−1	1	−1	1	$\mathbf{T_z}$	
E_1	2	$\sqrt{2}$	0	$-\sqrt{2}$	−2	0	0	$(\mathbf{T_x}, \mathbf{T_y})$	
E_2	2	0	−2	0	2	0	0		$(x^2 - y^2, xy)$
E_3	2	$-\sqrt{2}$	0	$\sqrt{2}$	−2	0	0	$(\mathbf{R_x}, \mathbf{R_y})$	(yz, zx)

D_{5d}	E	$2C_5$	$2C_5^2$	$5C_2$	i	$2S_{10}^3$	$2S_{10}$	$5\sigma_d$		
A_{1g}	1	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
A_{2g}	1	1	1	−1	1	1	1	−1	$\mathbf{R_z}$	
E_{1g}	2	2 cos 72°	2 cos 144°	0	2	2 cos 72°	2 cos 144°	0	$(\mathbf{R_x}, \mathbf{R_y})$	(yz, zx)
E_{2g}	2	2 cos 144°	2 cos 72°	0	2	2 cos 144°	2 cos 72°	0		$(x^2 - y^2, xy)$
A_{1u}	1	1	1	1	−1	−1	−1	−1		
A_{2u}	1	1	1	−1	−1	−1	−1	1	$\mathbf{T_z}$	
E_{1u}	2	2 cos 72°	2 cos 144°	0	−2	−2 cos 72°	−2 cos 144°	0	$(\mathbf{T_x}, \mathbf{T_y})$	
E_{2u}	2	2 cos 144°	2 cos 72°	0	−2	−2 cos 144°	−2 cos 72°	0		

D_{6d}	E	$2S_{12}$	$2C_6$	$2S_4$	$2C_3$	$2S_{12}^5$	C_2	$6C_2'$	$6\sigma_d$		
A_1	1	1	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
A_2	1	1	1	1	1	1	1	−1	−1	$\mathbf{R_z}$	
B_1	1	−1	1	−1	1	−1	1	1	−1		
B_2	1	−1	1	−1	1	−1	1	−1	1	$\mathbf{T_z}$	
E_1	2	$\sqrt{3}$	1	0	−1	$-\sqrt{3}$	−2	0	0	$(\mathbf{T_x}, \mathbf{T_y})$	
E_2	2	1	−1	−2	−1	1	2	0	0		$(x^2 - y^2, xy)$
E_3	2	0	−2	0	2	0	−2	0	0		
E_4	2	−1	−1	2	−1	−1	2	0	0		
E_5	2	$-\sqrt{3}$	1	0	−1	$\sqrt{3}$	−2	0	0	$(\mathbf{R_x}, \mathbf{R_y})$	(yz, zx)

The S_n groups

$\mathbf{S_4}$	E	S_4	C_2	S_4^3		
A	1	1	1	1	$\mathbf{R_z}$	$x^2 + y^2, z^2$
B	1	−1	1	−1	$\mathbf{T_z}$	$x^2 - y^2, xy$
E	{1	i	−1	$-i$}	$(\mathbf{T_x}, \mathbf{T_y}), (\mathbf{R_x}, \mathbf{R_y})$	(yz, zx)
	{1	$-i$	−1	i}		

$\mathbf{S_6}$	E	C_3	C_3^2	i	S_6^5	S_6		$\epsilon = \exp(2\pi i/3)$
A_g	1	1	1	1	1	1	$\mathbf{R_z}$	$x^2 + y^2, z^2$
E_g	{1	ϵ	ϵ^*	1	ϵ	ϵ^*}	$(\mathbf{R_x}, \mathbf{R_y})$	{$(x^2 - y^2, xy)$
	{1	ϵ^*	ϵ	1	ϵ^*	ϵ}		{(yz, zx)
A_u	1	1	1	−1	−1	−1	$\mathbf{T_z}$	
E_u	{1	ϵ	ϵ^*	−1	$-\epsilon$	$-\epsilon^*$}	$(\mathbf{T_x}, \mathbf{T_y})$	
	{1	ϵ^*	ϵ	−1	$-\epsilon^*$	$-\epsilon$}		

$\mathbf{S_8}$	E	S_8	C_4	S_8^3	C_2	S_8^5	C_4^3	S_8^7		$\epsilon = \exp(2\pi i/8)$
A	1	1	1	1	1	1	1	1	$\mathbf{R_z}$	$x^2 + y^2, z^2$
B	1	−1	1	−1	1	−1	1	−1	$\mathbf{T_z}$	
E_1	{1	ϵ	i	$-\epsilon^*$	−1	$-\epsilon$	$-i$	ϵ^*}	{$(\mathbf{T_x}, \mathbf{T_y})$	
	{1	ϵ^*	$-i$	$-\epsilon$	−1	$-\epsilon^*$	i	ϵ}	{$(\mathbf{R_x}, \mathbf{R_y})$	
E_2	{1	i	−1	$-i$	1	i	−1	$-i$}		$(x^2 - y^2, xy)$
	{1	$-i$	−1	i	1	$-i$	−1	i}		
E_3	{1	$-\epsilon^*$	$-i$	ϵ	−1	ϵ^*	i	$-\epsilon$}		(yz, xz)
	{1	$-\epsilon$	i	ϵ^*	−1	ϵ	$-i$	$-\epsilon^*$}		

The cubic groups

$\mathbf{T_d}$	E	$8C_3$	$3C_2$	$6S_4$	$6\sigma_d$		
A_1	1	1	1	1	1		$x^2 + y^2 + z^2$
A_2	1	1	1	−1	−1		
E	2	−1	2	0	0		$(2z^2 - x^2 - y^2, x^2 - y^2)$
T_1	3	0	−1	1	−1	$(\mathbf{R_x}, \mathbf{R_y}, \mathbf{R_z})$	
T_2	3	0	−1	−1	1	$(\mathbf{T_x}, \mathbf{T_y}, \mathbf{T_z})$	(xy, yz, zx)

$\mathbf{O_h}$	E	$8C_3$	$6C_2$	$6C_4$	$3C_2(=C_4^2)$	i	$6S_4$	$8S_6$	$3\sigma_h$	$6\sigma_d$		
A_{1g}	1	1	1	1	1	1	1	1	1	1		$x^2 + y^2 + z^2$
A_{2g}	1	1	−1	−1	1	1	−1	1	1	−1		
E_g	2	−1	0	0	2	2	0	−1	2	0		$(2z^2 - x^2 - y^2, x^2 - y^2)$
T_{1g}	3	0	−1	1	−1	3	1	0	−1	−1	$(\mathbf{R_x}, \mathbf{R_y}, \mathbf{R_z})$	
T_{2g}	3	0	1	−1	−1	3	−1	0	−1	1		(xy, yz, zx)
A_{1u}	1	1	1	1	1	−1	−1	−1	−1	−1		
A_{2u}	1	1	−1	−1	1	−1	1	−1	−1	1		
E_u	2	−1	0	0	2	−2	0	1	−2	0		
T_{1u}	3	0	−1	1	−1	−3	−1	0	1	1	$(\mathbf{T_x}, \mathbf{T_y}, \mathbf{T_z})$	
T_{2u}	3	0	1	−1	−1	−3	1	0	1	−1		

The $C_{\infty v}$ and $D_{\infty h}$ groups for linear molecules

$\mathbf{C_{\infty v}}$	E	$2C_\infty^\Phi$	. . .	$\infty\sigma_v$		
$A_1 = \Sigma+$	1	1	. . .	1	$\mathbf{T_z}$	$x^2 + y^2, z^2$
$A_2 = \Sigma-$	1	1	. . .	−1	$\mathbf{R_z}$	
$E_1 = \Pi$	2	$2\cos\Phi$	. . .	0	$(\mathbf{T_x}, \mathbf{T_y}), (\mathbf{R_x}, \mathbf{R_y})$	
$E_2 = \Delta$	2	$2\cos 2\Phi$	. . .	0		(yz, zx)
$E_3 = \Phi$	2	$2\cos 3\Phi$	. . .	0		$(x^2 - y^2, xy)$
. . .	. . .	. . .	. . .	. . .		

$\mathbf{D_{\infty h}}$	E	$2C_\infty^\Phi$	. . .	$\infty\sigma_v$	i	$2S_\infty^\Phi$	. . .	∞C_2		
Σ_g^+	1	1	. . .	1	1	1	. . .	1		$x^2 + y^2, z^2$
Σ_g^-	1	1	. . .	−1	1	1	. . .	−1	$\mathbf{R_z}$	
Π_g	2	$2\cos\Phi$	. . .	0	2	$-2\cos\phi$	. . .	0	$(\mathbf{R_x}, \mathbf{R_y})$	(yz, zx)
Δ_g	2	$2\cos 2\Phi$	. . .	0	2	$2\cos 2\Phi$	. . .	0		$(x^2 - y^2, xy)$
.	.	.	.	.	.	.	.	.		
.	.	.	.	.	.	.	.	.		
.	.	.	.	.	.	.	.	.		
Σ_u^+	1	1	. . .	1	−1	−1	. . .	−1	$\mathbf{T_z}$	
Σ_u^-	1	1	. . .	−1	−1	−1	. . .	1		
Π_u	2	$2\cos\Phi$	. . .	0	−2	$2\cos\Phi$	. . .	0	$(\mathbf{T_x}, \mathbf{T_y})$	
Δ_u	2	$2\cos 2\Phi$	. . .	0	−2	$-2\cos 2\Phi$	. . .	0		
. . .	. . .	. . .	. . .	. . .	. . .		. . .	. . .		

Multiplication properties of irreducible representations

General rules

A × A = A, B × B = A, A × B = B, A × E = E, B × E = E, A × T = T;
B × T = T;

g × g = g, u × u = g, u × g = u;
′ × ′ = ′, ″ × ″ = ′, ′ × ″ = ″;
$A \times E_1 = E_1$, $A \times E_2 = E_2$, $B \times E_1 = E_2$, $B \times E_2 = E_1$

Subscripts on A or B

1 × 1 = 1, 2 × 2 = 1, 1 × 2 = 2. except for $\mathbf{D}_2$ and $\mathbf{D}_{2h}$, where 1 × 2 = 3, 2 × 3 = 1, 1 × 3 = 2

Doubly-degenerate representations

For $\mathbf{C}_3$, $\mathbf{C}_{3h}$, $\mathbf{C}_{3v}$, $\mathbf{D}_3$, $\mathbf{D}_{3h}$, $\mathbf{D}_{3d}$, $\mathbf{C}_6$, $\mathbf{C}_{6h}$, $\mathbf{C}_{6v}$, $\mathbf{D}_6$, $\mathbf{D}_{6h}$, $\mathbf{S}_6$, $\mathbf{O}_h$, $\mathbf{T}_d$,:

$$E_1 \times E_1 = E_2 \times E_2 = A_1 + A_2 + E_2$$
$$E_1 \times E_2 = B_1 + B_2 + E_1$$

For $\mathbf{C}_4$, $\mathbf{C}_{4v}$, $\mathbf{C}_{4h}$, $\mathbf{D}_{2d}$, $\mathbf{D}_4$, $\mathbf{D}_{4h}$, $\mathbf{S}_4$:

$$E \times E = A_1 + A_2 + B_1 + B_2$$

For groups in above lists that have symbols A, B or E without subscripts, read $A_1 = A_2 = A$, etc.

Triply-degenerate representations:

For $\mathbf{T}_d$, $\mathbf{O}_h$:

$$E \times T_1 = E \times T_2 = T_1 + T_2$$
$$T_1 \times T_1 = T_2 \times T_2 = A_1 + E + T_1 + T_2$$
$$T_1 \times T_2 = A_2 + E + T_1 + T_2$$

Linear molecules ($\mathbf{C}_{\infty v}$ and $\mathbf{D}_{\infty h}$):

$\Sigma^+ \times \Sigma^+ = \Sigma^- \times \Sigma^- = \Sigma^+$; $\Sigma^+ \times \Sigma^- = \Sigma^-$
$\Sigma^+ \times \Pi = \Sigma^- \times \Pi = \Pi$; $\Sigma^+ \times \Delta = \Sigma^- \times \Delta = \Delta$; etc.
$\Pi \times \Pi = \Sigma^+ + \Sigma^- + \Delta$
$\Delta \times \Delta = \Sigma^+ + \Sigma^- + \Gamma$
$\Pi \times \Delta = \Pi + \Phi$

Contribution to character per unshifted atom

R	$\chi(R)$
E	+3
i	−3
σ	+1
C_2	−1
C_3^1, C_3^2	0
C_4^1, C_4^3	+1
C_6^1, C_6^5	+2
S_3^1, S_3^5	−2
S_4^1, S_4^3	−1
S_6^1, S_6^5	0