Schrodinger Equation (zero potential)

$$-\frac{\hbar^2}{2I}\frac{d^2}{d\phi^2}\Phi(\phi) = E\Phi(\phi)$$

$$-\frac{\hbar^2}{2I}\Lambda^2 Y(\theta,\phi) = EY(\theta,\phi)$$

Hydrogenic Laplacian

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\Lambda^2$$

$$\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} = \frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r} = \frac{1}{r}\frac{\partial^2}{\partial r^2}r$$

Legendrian

$$\Lambda^2 = \frac{1}{sin^2\theta}\frac{\partial^2}{\partial \phi^2} + \frac{1}{sin\theta}\frac{\partial}{\partial \theta}sin\theta\frac{\partial}{\partial \theta}$$

Angular wavefunctions

$$\Phi(\phi) = Ae^{+im_l\phi} \qquad m_l = 0, \pm 1, \pm 2 \ldots$$

$$\Theta(\theta) = \left\{\left(\frac{2l+1}{2}\right)\frac{(l-|m_l|!)}{(l+|m_l|!)}\right\}^{\frac{1}{2}} P_l^{|m_l|}(cos\theta)$$

Associated Legendre Functions

$$P_0^0(\cos\theta) = 1$$
$$P_1^0(\cos\theta) = \cos\theta$$
$$P_1^1(\cos\theta) = -\sin\theta$$
$$P_2^0(\cos\theta) = \tfrac{1}{2}\left(3\cos^2\theta - 1\right)$$
$$P_2^1(\cos\theta) = -3\sin\theta\cos\theta$$
$$P_2^2(\cos\theta) = 3\sin^2\theta$$
$$P_3^0(\cos\theta) = \tfrac{1}{2}\cos\theta\left(5\cos^2\theta - 3\right)$$
$$P_3^1(\cos\theta) = -\tfrac{3}{2}\left(5\cos^2\theta - 1\right)\sin\theta$$
$$P_3^2(\cos\theta) = 15\cos\theta\sin^2\theta$$
$$P_3^3(\cos\theta) = -15\sin^3\theta.$$

Spherical Harmonics

$$Y_0^0(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{1}{\pi}}$$
$$Y_1^{-1}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{2\pi}}\sin\theta\, e^{-i\varphi}$$
$$Y_1^0(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{\pi}}\cos\theta$$
$$Y_1^1(\theta,\varphi) = \frac{-1}{2}\sqrt{\frac{3}{2\pi}}\sin\theta\, e^{i\varphi}$$
$$Y_2^{-2}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{-2i\varphi}$$
$$Y_2^{-1}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{15}{2\pi}}\sin\theta\cos\theta\, e^{-i\varphi}$$
$$Y_2^0(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{5}{\pi}}(3\cos^2\theta - 1)$$
$$Y_2^1(\theta,\varphi) = \frac{-1}{2}\sqrt{\frac{15}{2\pi}}\sin\theta\cos\theta\, e^{i\varphi}$$
$$Y_2^2(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\varphi}$$

Hydrogenic Radial Equations, Generalised Leguarre Functions

$$R_{10} = \left(\frac{Z}{a}\right)^{3/2} 2e^{-\rho/2}$$
$$R_{20} = \left(\frac{Z}{a}\right)^{3/2}\left(\frac{1}{8}\right)^{1/2}(2-\rho)e^{-\rho/2}$$
$$R_{21} = \left(\frac{Z}{a}\right)^{3/2}\left(\frac{1}{24}\right)^{1/2}\rho e^{-\rho/2}$$
$$R_{30} = \left(\frac{Z}{a}\right)^{3/2}\left(\frac{1}{243}\right)^{1/2}(6-6\rho+\rho^2)e^{-\rho/2}$$
$$R_{31} = \left(\frac{Z}{a}\right)^{3/2}\left(\frac{1}{486}\right)^{1/2}(4-\rho)\rho e^{-\rho/2}$$
$$R_{32} = \left(\frac{Z}{a}\right)^{3/2}\left(\frac{1}{2430}\right)^{1/2}\rho^2 e^{-\rho/2}$$

Real and Imaginary orbitals

$$p_0 = p_z = \left(\frac{3}{4\pi}\right)^{1/2} R_{n,1}(r)cos\theta$$
$$p_{+1} = -\left(\frac{3}{8\pi}\right)^{1/2} R_{n,1}(r)sin\theta e^{+i\phi}$$
$$p_{-1} = \left(\frac{3}{8\pi}\right)^{1/2} R_{n,1}(r)sin\theta e^{-i\phi}$$
$$p_x = \frac{1}{\sqrt{2}}(p_{-1} - p_{+1}) = \left(\frac{3}{4\pi}\right)^{1/2} R_{n,1}(r)sin\theta cos\phi$$
$$p_y = \frac{i}{\sqrt{2}}(p_{-1} - p_{+1}) = \left(\frac{3}{4\pi}\right)^{1/2} R_{n,1}(r)sin\theta sin\phi$$