

## 423 Molecular Orbital Theory Assignment

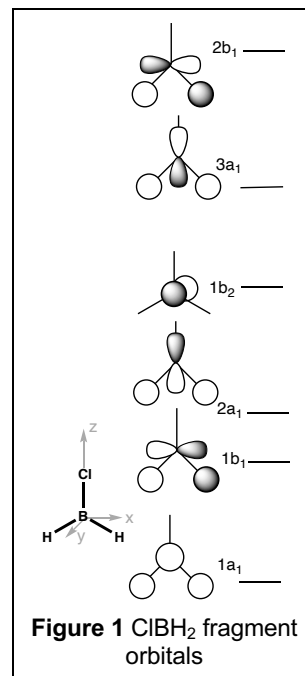
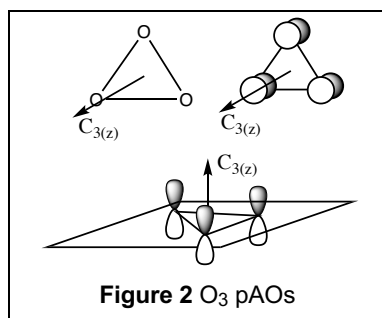
Set: 13th March 2026. Due: **25th March 5pm**

*Hand-in as a PDF file on Nuku (the Assignment in the 423 folder). Diagrams and text need to be legible to me and photographs/scans clear please!*

**Q1.** Draw a MO diagram for  $C_{2v}$  ClBH<sub>2</sub>. Use the BH<sub>2</sub> fragment orbitals given in **Figure 1**. The Cl pAOs will have an energy between the 1b<sub>1</sub> and 2a<sub>1</sub> FOs and the Cl sAO will interact with the 1a<sub>1</sub> FO. Explicitly demonstrate MO mixing by drawing out and combining the relevant MOs, discuss MO mixing with respect to these MOs. Depict the mixed MOs on your MO diagram.

15 marks

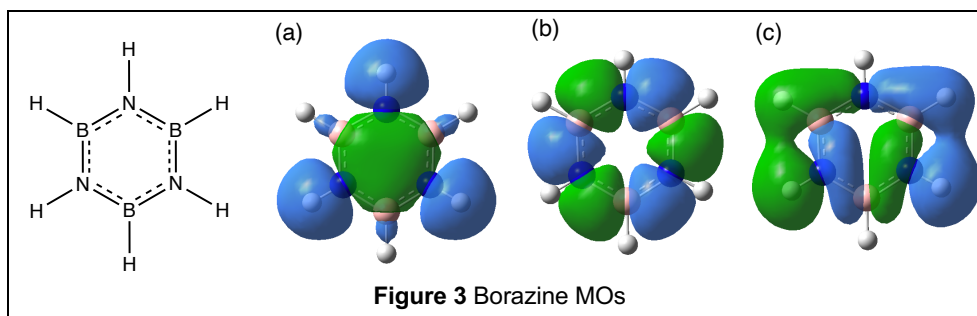
**Q2** Consider three oxygen atoms arranged in an equilateral triangle, that belong to the  $D_{3h}$  point group, **Figure 2**. In the following show your working at all stages.



- i) Determine the reducible representation for the three  $p_\pi$  orbitals
- ii) Write down the reduction and projection formulae. Briefly explain the terms in both formulae.
- iii) Use the reduction formula to determine the symmetry of the  $p_\pi$  based symmetry adapted fragment orbitals.
- iv) Use the projection formula to determine the wave function of the  $p_\pi$  based symmetry adapted fragment orbitals.
- v) Draw an energy level diagram for the  $p_\pi$  based symmetry adapted fragment orbitals. Draw the fragment orbitals and write the associated equation beside each energy level. Label the symmetry of each orbital.

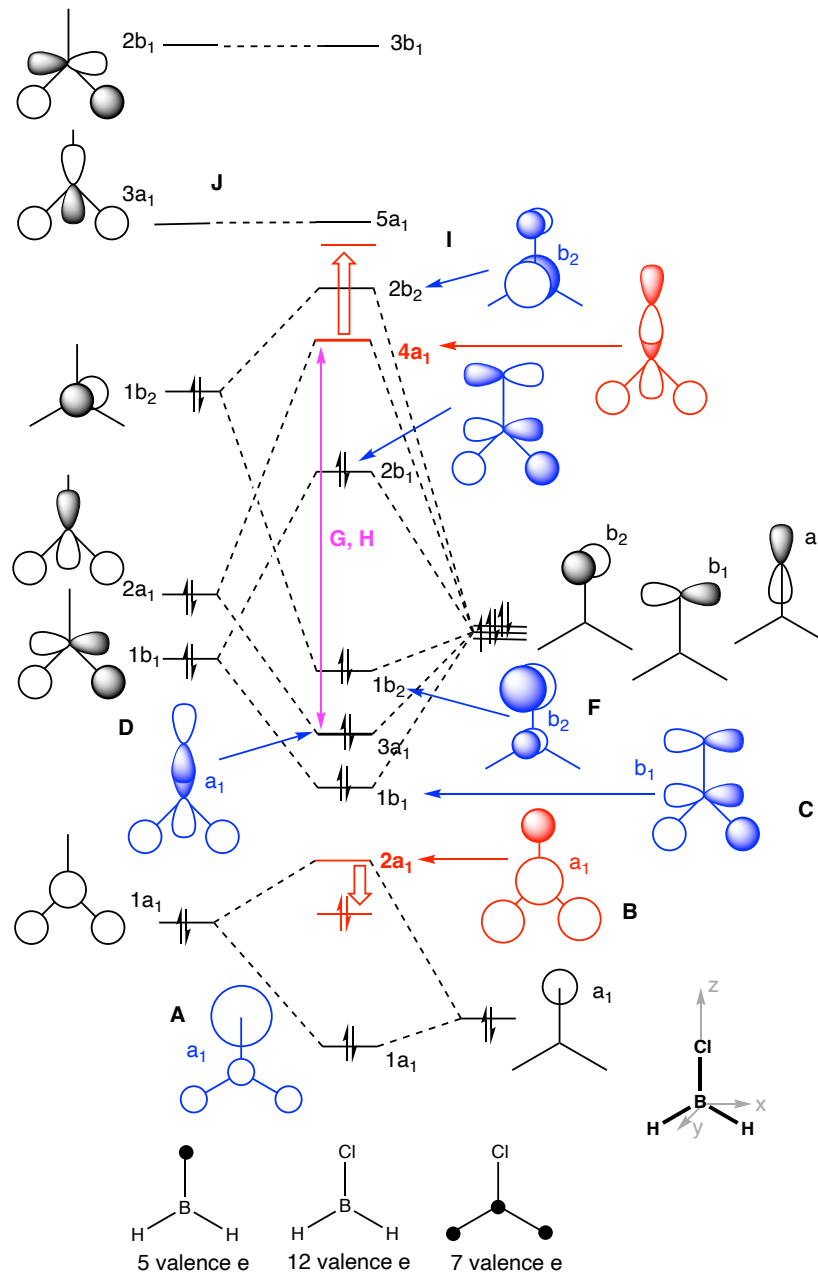
10 marks

**Q3** Draw LCAOs for the computed MOs of borazine shown in **Figure 3**. On your diagrams annotate features important for evaluating the MO bonding character.



10 marks

**Q1.** Draw a MO diagram for  $C_{2v}$  ClBH<sub>2</sub>. Use the BH<sub>2</sub> fragment orbitals given in **Figure 1**. The Cl pAOs will have an energy between the 1b<sub>1</sub> and 2a<sub>1</sub> FOs and the Cl sAO will interact with the a<sub>1</sub> FO. Explicitly demonstrate MO mixing by drawing out and combining the relevant MOs, discuss MO mixing with respect to these MOs. Depict the mixed MOs on your MO diagram. 15 marks



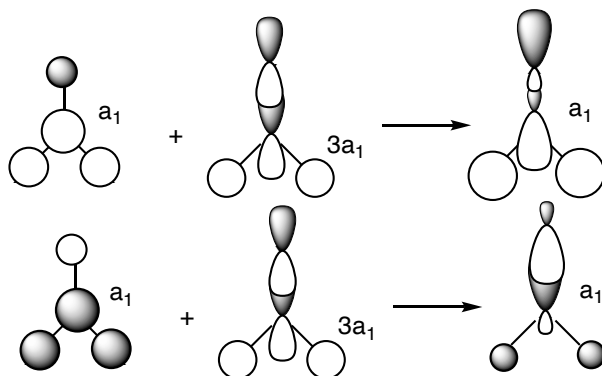
- annotations!
  - A** the 1a<sub>1</sub> a larger contribution from the Cl sAO as this AO is lower in energy and this MO is the bonding combination
  - B** the 2a<sub>1</sub> a larger contribution from the BH<sub>2</sub> fragment orbital as this FO is higher in energy and this MO is the anti-bonding combination
  - C** Cl is electronegative and on the far right of the PT, it will have a large sp-gap
  - D** the differences in FO contributions are minor for the 1b<sub>1</sub>, 3a<sub>1</sub> and 1b<sub>2</sub> because the FOs are all very close in energy
  - E** the energy ordering of the 1b<sub>1</sub> and 3a<sub>1</sub> is difficult to predict, the FO for b<sub>1</sub> lie lower in energy but the FOs for a<sub>1</sub> have better overlap
  - F** the 1b<sub>2</sub> will be higher in energy due to the "side on" or π overlap of the FOs
  - G** antibonding MOs are destabilised by more than bonding MOs are stabilised

**H** the  $1b_1$  and  $2b_1$  will have a smaller splitting energy because these MOs have side-on overlap while the  $3a_1$  and  $4a_1$  will have a larger splitting energy because these MOs have directed p-p overlap meaning the interaction will be stronger

**I** the  $1b_2$  and  $2b_2$  have different fragment coefficients because the contributing FOs are further apart in energy, the higher energy  $BH_2$   $b_1$  contributes most to the antibonding MO,  $2b_2$

**J** the higher energy  $BH_2$  FOs remain non-bonding

- necessary conditions for MO mixing to occur:
  - only MOs of the same symmetry can mix
  - MOs cannot be from the same bonding-antibonding pair
  - mixing must stabilise the total energy of the molecule
- mixing tends to be large or important when:
  - MOs are close in energy
  - one of the MOs is non-bonding or unoccupied
  - MOs are in the HOMO-LUMO region
  - MOs that mix form a new bonding/antibonding pair.
  - the closer in energy the stronger mixing between MOs
- in this case the options are the  $2b_1$  and  $3b_1$  but they belong to the same  $BH_2$  FO bonding and antibonding pair, and are far apart in energy
- the  $3a_1$  and  $4a_1$  belong to the same MO bonding and antibonding pair
- the other option is the  $2a_1$  and  $4a_1$  MOs, while the  $2a_1$  is occupied the  $4a_1$  is unoccupied, the energy difference is large so any splitting will be small.
- the mixing is shown below the top orbital is made less antibonding and the bottom orbital is significantly more antibonding.



MO diagram marks:

0.5 axial system defined in correct alignment and placed on the molecule

0.5 fragments and molecule along the bottom, placeholders present

1 energies, depicting and labelling FOs

3 for shapes of MOs, including size variations in MOs

1 correct symmetry labels MOs

2 MO energies roughly right

1 electronic configuration

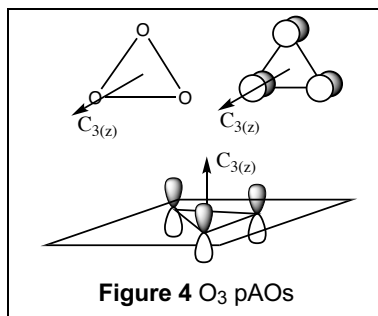
3 for correct mixing MOs, rules for mixing

3 for a selection of annotations related to MOs and energies

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15 marks

**Q2** Consider three oxygen atoms arranged in an equilateral triangle, that belong to the  $D_{3h}$  point group, **Figure 2**. In the following show your working at all stages.



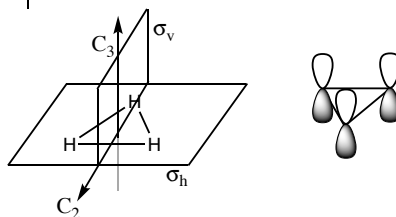
find the character table

$D_{3h}$	$E$	$2C_3$	$3C_2$	$\sigma_h$	$2S_3$	$\sigma_v$		
$A_1'$	1	1	1	1	1	1		$x^2 + y^2, z^2$
$A_2'$	1	1	-1	1	1	-1	$R_z$	
$E'$	2	-1	0	2	-1	0	$(T_x, T_y)$	$x^2 - y^2, xy$
$A_1''$	1	1	1	-1	-1	-1		
$A_2''$	1	1	-1	-1	-1	1	$T_z$	
$E''$	2	-1	0	-2	1	0	$(R_x, R_y)$	$xz, yz$

i) Determine the reducible representation for the three  $p_\pi$  orbitals

- use all three  $p_z$  orbitals as a basis
- determine the number of orbitals that don't shift from one atomic center to another under the symmetry operations of the group, atoms that move contribute 0, for the atoms that move if the phase is unchanged an orbital contributes +1, if the phase changes the orbital contributes (-1).

$D_{3h}$	$E$	$2C_3$	$3C_2$	$\sigma_h$	$2S_3$	$3\sigma_v$
$\Gamma(3p_z)$	3	0	-1	-3	0	1



selected elements from the  $D_{3h}$  point group

ii) Write down the reduction and projection formulae. Briefly explain the terms in both formulae.

- this is straight from the notes, please see the relevant section

### The Reduction formula

$$n_{IR} = \frac{1}{h} \sum_Q k \cdot \chi^{IR}(Q) \cdot \chi^R(Q)$$

$h$  = number of operations in the group

$Q$  = a particular symmetry operation

$k$  = the number of operations of  $Q$

$\chi^{IR}(Q)$  the character of the Irreducible Representation under  $Q$

$\chi^R(Q)$  = the character of the Reducible Representation under  $Q$

### The Projection Operator

$$P_{\Gamma}[\psi] = \frac{1}{h} \sum_Q \chi^{IR}(Q) \cdot Q[\psi]$$

h = number of operations in the group

Q = a particular symmetry operation

$P_{\Gamma}[\Psi]$  = operate on an orbital function

$\chi^{IR}(Q)$  = the character of the Irreducible Representation under Q

iii) Use the reduction formula to determine the symmetry of the  $p_{\pi}$  based symmetry adapted fragment orbitals.

- Be smart! do not just go through all the IR sequentially!
- look at the character table and look for the IR that will give the all in-phase combination
  - (a) this cannot be  $a_1'$  since under  $\sigma_h$  the pAOs will invert giving -1 character
  - (b) under  $\sigma_h$  the reducible rep has -3 so likely to be  $a_1''$  or  $a_2''$  or  $e''$
  - (c) try highest symmetry one first =>  $a_1''$

$D_{3h}$	E	$2C_3$	$3C_2$	$\sigma_h$	$2S_3$	$3\sigma_v$
$A_1''$	1	1	1	-1	-1	-1
$\Gamma(3p_z)$	3	0	-1	-3	0	1

$$n_{A_1''} = \frac{1}{12} \left[ (1 \cdot 1 \cdot 3) + (2 \cdot 1 \cdot 0) + (3 \cdot 1 \cdot -1) + (1 \cdot -1 \cdot -3) + (2 \cdot -1 \cdot 0) + (3 \cdot -1 \cdot 1) \right]$$

$$n_{A_1''} = \frac{1}{12} \left[ 3 + 0 - 3 + 3 + 0 - 3 \right] = 0$$

- this hasn't worked, so try the next one  $a_2''$

$D_{3h}$	E	$2C_3$	$3C_2$	$\sigma_h$	$2S_3$	$3\sigma_v$
$A_2''$	1	1	-1	-1	-1	1
$\Gamma(3p_z)$	3	0	-1	-3	0	1

$$n_{A_2''} = \frac{1}{12} \left[ (1 \cdot 1 \cdot 3) + (2 \cdot 1 \cdot 0) + (3 \cdot -1 \cdot -1) + (1 \cdot -1 \cdot -3) + (2 \cdot -1 \cdot 0) + (3 \cdot 1 \cdot 1) \right]$$

$$n_{A_2''} = \frac{1}{12} \left[ 3 + 0 + 3 + 3 + 0 + 3 \right] = \frac{12}{12} = 1$$

- now we know that there is 1  $a_2''$  component we subtract this from the reducible representation

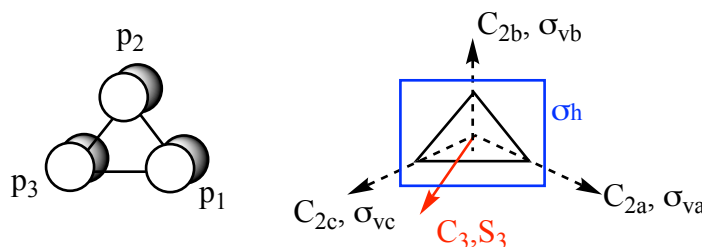
$D_{3h}$	E	$2C_3$	$3C_2$	$\sigma_h$	$2S_3$	$3\sigma_v$
$\Gamma(3p_z)$	3	0	-1	-3	0	1
$A_2''$	1	1	-1	-1	-1	1
$\Gamma(3p_z) - A_2''$	2	-1	0	-2	1	0
				= $E''$		

- this tells us that  $\Gamma(3p_z) = a_2'' + e''$
- you should also do a check by adding the two IR and show you regenerate  $\Gamma(3p_z)$

$D_{3h}$	E	$2C_3$	$3C_2$	$\sigma_h$	$2S_3$	$3\sigma_v$
$A_2''$	1	1	-1	-1	-1	1
$E''$	2	-1	0	-2	1	0
-----						
$A_2'' + E''$	3	0	-1	-3	0	1

$=\Gamma(3p_z)$

iv) Use the projection formula to determine the wave function of the  $p_\pi$  based symmetry adapted fragment orbitals.



$D_{3h}$	E	$C_3^1$	$C_3^2$	$C_{2a}$	$C_{2b}$	$C_{2c}$	$\sigma_h$	$S_3^1$	$S_3^{-1}$	$\sigma_{va}$	$\sigma_{vb}$	$\sigma_{vc}$
$Q[p_1]$	$p_1$	$p_2$	$p_3$	$-p_1$	$-p_3$	$-p_2$	$-p_1$	$-p_2$	$-p_3$	$p_1$	$p_3$	$p_2$
$A_2''$	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1
$\chi^{A_2''} Q[p_1]$	$p_1$	$p_2$	$p_3$	$p_1$	$p_3$	$p_2$	$p_1$	$p_2$	$p_3$	$p_1$	$p_3$	$p_2$

$$P_{A_2''}[p_1] = \frac{1}{12}[p_1 + p_2 + p_3 + p_1 + p_3 + p_2 + p_1 + p_2 + p_3 + p_1 + p_3 + p_2]$$

$$= \frac{1}{12}[4p_1 + 4p_2 + 4p_3]$$

$$= \frac{1}{3}[p_1 + p_2 + p_3]$$

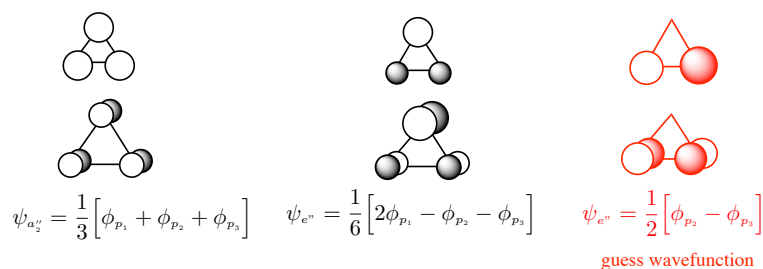
$D_{3h}$	E	$C_3^1$	$C_3^2$	$C_{2a}$	$C_{2b}$	$C_{2c}$	$\sigma_h$	$S_3^1$	$S_3^{-1}$	$\sigma_{va}$	$\sigma_{vb}$	$\sigma_{vc}$
$Q[p_1]$	$p_1$	$p_2$	$p_3$	$-p_1$	$-p_3$	$-p_2$	$-p_1$	$-p_2$	$-p_3$	$p_1$	$p_3$	$p_2$
$E''$	2	-1	-1	0	0	0	-2	1	1	0	0	0
$\chi^{E''} Q[p_1]$	$2p_1$	$-p_2$	$-p_3$	0	0	0	$2p_1$	$-p_2$	$-p_3$	0	0	0

$$P_{E_2}[p_1] = \frac{1}{12}[2p_1 - p_2 - p_3 + 2p_1 - p_2 - p_3]$$

$$= \frac{1}{12}[4p_1 - 2p_2 + 2p_3]$$

$$= \frac{1}{6}[2p_1 - p_2 - p_3]$$

- now you need to guess an orthogonal partner, you already know the orbital shapes for the  $H_3$  system



- if you look directly down from above on the pAO system it looks just like the sAO system, (except that in the pAO system reflection in the  $\sigma_h$  plane alters the phase!). The first two orbitals have already been derived, we guess that the third will look like the last sAO MO

but with a node in sh plane and a change of phase. Then we need to check that the two e'' MOs are orthogonal: which is true!

- we have the two equations

$$\psi_{1e''}^1 = 2\phi_{p1} - \phi_{p2} - \phi_{p3}$$

$$\psi_{1e''}^2 = \phi_{p2} - \phi_{p3}$$

- now show they are orthogonal

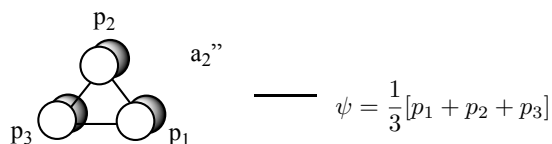
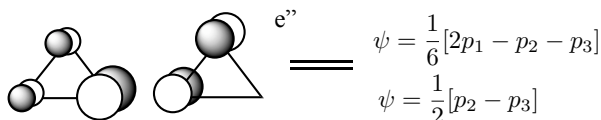
$$\begin{aligned} \int \psi_{1e''}^1 \cdot \psi_{1e''}^2 d\tau &= \int (2\phi_{p1} - \phi_{p2} - \phi_{p3}) \cdot (\phi_{p2} - \phi_{p3}) d\tau \\ &= \underbrace{\int 2\phi_{p1}\phi_{p2} d\tau}_{=2s} - \underbrace{\int 2\phi_{p1}\phi_{p3} d\tau}_{=2s} - \underbrace{\int \phi_{p2}\phi_{p2} d\tau}_{=1} + \underbrace{\int \phi_{p2}\phi_{p3} d\tau}_{=s} - \underbrace{\int \phi_{p3}\phi_{p2} d\tau}_{=s} + \underbrace{\int \phi_{p3}\phi_{p3} d\tau}_{=1} \\ &= 2s - 2s - 1 + s - s + 1 \\ &= 0 \end{aligned}$$

- where we have used the fact that the atomic orbitals are normalised

$$\int \phi_i \cdot \phi_j d\tau = s_{ij} \quad \text{and} \quad \int \phi_i \cdot \phi_i d\tau = 1$$

- and that the overlap between the orbitals is the same because the orbitals are all the same and they are all equally distant from each other ie  $s_{12}=s_{21}=s_{13}=s_{23}$

- v) Draw an energy level diagram for the  $p_\pi$  based symmetry adapted fragment orbitals. Draw the fragment orbitals and write the associated equation beside each energy level. Label the symmetry of each orbital.



10 marks

Symmetry adapted FOs marks:

1 reducible representation

1 definitions

2 determining nIR

1 projection p1

2 projection tables

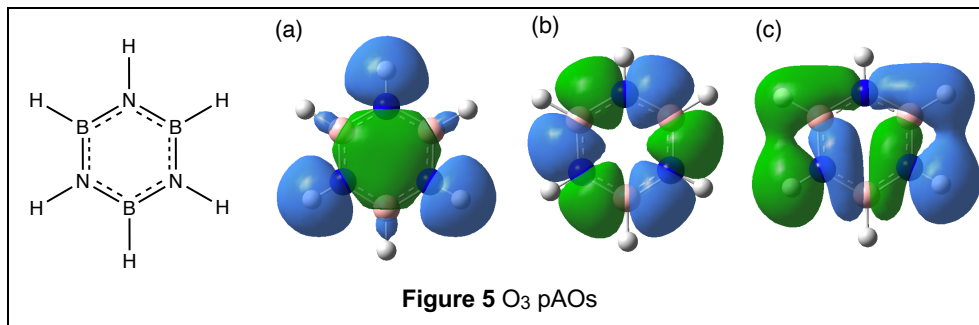
2 orthogonality

1 final diagram

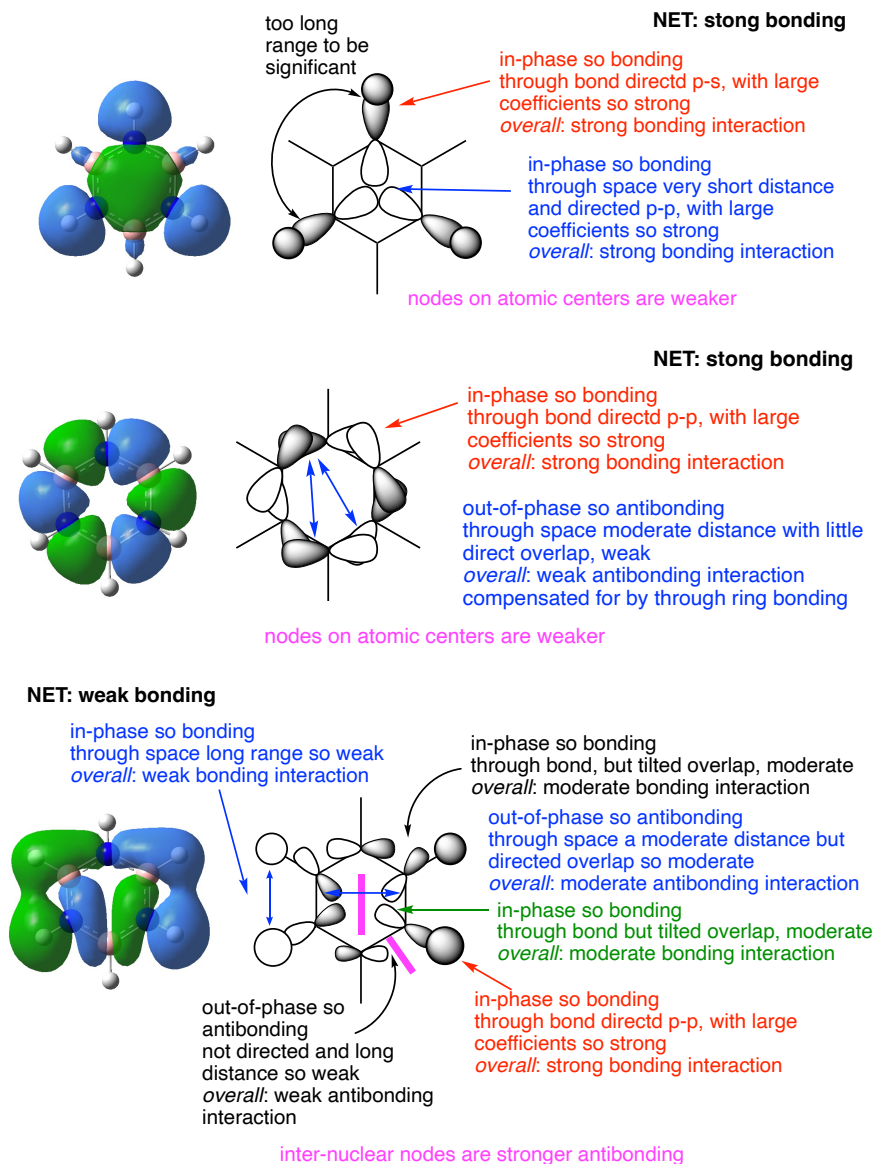
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10 marks

**Q3** Draw LCAOs for the computed MOs of borazine shown in **Figure 3**. On your diagrams annotate features important for evaluating the MO bonding character.



10 marks



LCAOs from MOs marks:

3 MO1

3 MO2

4 MO3

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10 marks